IIR lifting DWT for Lossless Image Compression

Oleksiy Pogrebnyak[1], Pablo Manrique Ramírez[2], Luis Pastor Sanchez Fernandez[3],

Instituto Politecnico Nacional, CIC-IPN, Av.Juan de Dios Batiz s/n,
Colonia Nueva Industrial Vallejo, C.P. 07738, Mexico D.F.
E-mail: [1]olek@pollux.cic.ipn.mx, [2]pmanriq@cic.ipn.mx, [3]lsanchez@cic.ipn.mx

Abstract. A data dependent wavelet transform based on the modified lifting scheme is presented. The algorithm is based on the wavelet filters derived from a generalized lifting scheme. The proposed framework for the lifting scheme permits to obtain easily different wavelet FIR filter coefficients in the case of the (~N, N) lifting and improve the performance by additional IIR filtering of the already calculated wavelet coefficients. The perfect image restoration in this case is obtained employing the particular features of the lifting scheme. Changing wavelet IIR filter coefficients, one can obtain the filter frequency response that match better to the image data than the standard lifting filters, resulting in higher data compression rate. The designed algorithm was tested on different images. The obtained simulation results show that the proposed method performs better in data compression for various images in comparison to the standard technique resulting in significant savings in compressed data length.

Key words: image processing, wavelets, lifting scheme, adaptive compression

1 Introduction

In the past decade, the wavelet transform has become a popular, powerful tool for different image and signal processing applications such as noise cancellation, data compression, feature detection, etc. Meanwhile, the aspect of wavelet decomposition/reconstruction implementation, especially for image compression applications, now continues to be under consideration.

The first algorithm of the fast discrete wavelet transform (DWT) was proposed by S.G.Mallat [1]. This algorithm is based on the fundamental work of Vetterli [2] on signal/image subband decomposition by 1-D quadrature-mirror filters (QMF), and orthonormal wavelet bases proposed by I.Daubechies [3]. Then, W.Sweldens [4] proposed the lifting scheme based on polyphase factorization of known wavelets that now is widely used (for example, in JPEG2000 standard) for lossless image/signal compression based on DWT. To enhance the energy compaction characteristics of the DWT, different methods basing on an adaptive lifting scheme [4 - 8], principal components filter banks [9] and signal-dependent wavelet/subband filter banks [10-12] were developed recently.

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In this paper, we present an algorithm for lossless image compression that is based on a subclass of IIR wavelet filters. These filters are derived from the generalized FIR wavelet lifting filters [8, 13, 14] introducing zeros in the prototype FIR filters. Performing causal filtering at the analysis and anticausal filtering of the time-inverted data at the synthesis stages, one can obtain the perfect image restoration with the presented IIR filters. Varying the order of the filter and the filter coefficients depending on the image data statistical/spectral properties, the decompositions can be optimized to achieve a minimum of the entropy in the wavelet domain.

2 DWT by Generalized Lifting Scheme

The lifting scheme [3] is widely used in the wavelet based image analysis. Its main advantages are: the reduced number of calculations; less memory requirements; the possibility of the operation with integer numbers for lossless data compression. The lifting scheme consists of the following basic operations: splitting, prediction and update.

Splitting is sometimes referred to as the lazy wavelet. This operation splits the original signal $\vec{x}$ into odd and even samples:

$$s_i = x_{2i}, \quad d_i = x_{2i+1}. \tag{1}$$

Prediction, or the dual lifting at the level k calculates the wavelet coefficients, or the details $\{d^{(k)}\}$ as the error of prediction of $\{d^{(k-1)}\}$ on $\{s^{(k-1)}\}$[14]:

$$d_i^{(k)} = d_i^{(k-1)} + \sum_{j=-\tilde{N}}^{\tilde{N}} b_j^p \cdot s_{i+j}^{(k-1)}, \tag{2}$$

where $\vec{b}^p$ are coefficients of the wavelet-based high-pass FIR filter and $\tilde{N}$ is the prediction filter order.

Update, or the primal lifting combines $\vec{s}^{(k-1)}$ and $\vec{d}^{(k)}$, and consists of low-pass FIR filtering to obtain a coarse approximation of the original signal $\vec{x}$:

$$s_i^{(k)} = s_i^{(k-1)} + \sum_{j=-N}^{N} b_j^u \cdot d_{i+j}^{(k)}, \tag{3}$$

where $\vec{b}^u$ are coefficients of the wavelet-based low-pass FIR filter and N is the update filter order.

The inverse transform is straightforward: first, the signs of FIR filter coefficients $\{u\}$ and $\{p\}$ are switched; the inverse update followed by inverse prediction is calculated. Finally, the odd and even data samples are merged.

In [13], the FIR filters that participate in the prediction and update operation were represented in the domain of Z-transform. Using this approach, we formulated the transfer function of the prediction FIR filter as follows [8, 14]:

$$H_p(z) = 1 + b_0^p\left(z + z^{-1}\right) + b_1^p\left(z^3 + z^{-3}\right) + \ldots + b_{\tilde{N}-1}^p\left(z^{2\tilde{N}-1} + z^{-2\tilde{N}+1}\right), \tag{4}$$

The $H_p(z)$ must has zero at $\omega = 0$, i.e., at $z = 1$. It can be easily found [5] that this condition is satisfied when

$$\sum_{i=0}^{\tilde{N}-1} b_i^p = -\frac{1}{2}. \tag{5}$$

When the condition (5) is satisfied, $H_p(-1) = 2$ and $H_p(0) = 1$ that means the prediction filter has gain 2 at $\omega = \pi$ and unit gain at $\omega = \frac{\pi}{2}$.

Following this approach, the transfer function for update filter can be obtained in the terms of $H_p(z)$ [8, 14]:

$$H_u(z) = 1 + H_p(z)\left\{b_0^u\left(z + z^{-1}\right) + b_1^u\left(z^3 + z^{-3}\right) + \ldots + b_{N-1}^u\left(z^{2N-1} + z^{-2N+1}\right)\right\}. \tag{6}$$

Similarly, $H_u(z)$ must has zero at $\omega = \pi$, i.e., at $z = -1$. It can be easily found [8, 13] that this condition is satisfied when

$$\sum_{i=0}^{N-1} b_i^u = \frac{1}{4}. \tag{7}$$

When the condition (7) is satisfied, $H_u(1) = 1$ and that means the prediction filter has gain 1 at $\omega = 0$.

Using the generalization of the lifting scheme (4)- (7) and a simplified lifting coefficients' representation [13], we proposed in [14] the following recursive representation for lifting FIR filter coefficients of order $2\tilde{N}$

$$b_0^p = -\frac{128 + B_p}{256},\ b_1^p = \frac{B_p}{256} - b_2^p,\ b_i^p = -\frac{b_{i-1}^p}{C_p} - b_{i+1}^p \tag{8}$$

$$b_0^u = \frac{64 - B_u}{256},\ b_1^u = \frac{B_u}{256} - b_2^u,\ b_i^u = -\frac{b_{i-1}^u}{C_u} - b_{i+1}^u \tag{9}$$

where B_p, B_u, C_p, C_u are the parameters that control the DWT properties. The correspondences between these control parameters B_p, B_u,and the conventional (non-lifted) biorthogonal wavelet filters can be found in reference [13]. In the formulas (8), (9) the parameters B_p, B_u control the width of the transition bands and the parameters C_p, C_u control the smoothness of the pass and stop bands to prevent the appearance of the lateral lobes: with greater values of B_p, B_u the values of

C_p, C_u tend to be greater [8]. In practice, one can use predictor (4) and update filter (6) with $\tilde{N}=6$, $N=6$, $B_p=20, B_u=8$, $C_p=6, C_u=6$ to achieve narrow transition bands and good compression rate [8, 14].

3 IIR Lifting Scheme

Considering generalized lifting scheme (4), (6) that these all-zeros systems can be modified to obtain rational transfer functions of a special form containing zeros and poles as following [14]:

$$H_p(z)=\frac{1+b_0^p\left(z+z^{-1}\right)+b_1^p\left(z^3+z^{-3}\right)+\ldots+b_{\tilde{N}-1}^p\left(z^{2\tilde{N}-1}+z^{-2\tilde{N}+1}\right)}{1+a_2^p z^{-2}+a_4^p z^{-4}}, \tag{10}$$

$$H_u(z)=\frac{1+H_p(z)\left\{b_0^u\left(z+z^{-1}\right)+b_1^u\left(z^3+z^{-3}\right)+..+b_{N-1}\left(z^{2N-1}+z^{-2N+1}\right)\right\}}{1+a_2^u z^{-2}+a_4^u z^{-4}}. \tag{11}$$

A specific condition to lifting predictor is that it must have a fixed gain to fulfill condition (7), i.e., to prevent bias in the output of the update filer at $\omega=0$. This can be done introducing normalization by factor $1-a_{2p}-a_{4p}-\ldots$ in (11) [14]:

$$H_u(z)=\frac{1+H_p(z)\left\{u_0\left(z+z^{-1}\right)+u_1\left(z^3+z^{-3}\right)+..+u_{N-1}\left(z^{2N-1}+z^{-2N+1}\right)\right\}}{\left(1-a_{2p}-a_{4p}\right)\left(1+a_{2u}z^{-2}+a_{4u}z^{-4}\right)}. \tag{12}$$

Another problem arises when implementing inverse transform with IIR lifting. The wavelet analysis/synthesis filters must provide the perfect restoration of the original data that is especially important for lossless data compression. In the traditional dyadic wavelet decompositions/restorations technique, special care is took to design orthonormal filter banks where each filter satisfies Nyquist constraint $\left|H_k\left(e^{j\omega}\right)\right|_{\downarrow 2}=1$ [9]. In difference, the lifting scheme has a potential to design biorthogonal IIR wavelet filters in simpler way: in the restoration stage, one can use inverse predictor and inverse update filter that operates upon rearranging the input signal elements (wavelet coefficients) backward and then filtering them with the inverse filters for synthesis and next time performing rearranging of the data $\{\cdot\}^B$ [14].

Next, we want to proceed with integer calculus whereas it is possible. For this purpose, we use the representation of normalized coefficients a_i^p, a_i^u [14]:

$$a_i^p=\frac{A_i^p}{256},\ a_i^u=\frac{A_i^u}{256}. \tag{26}$$

Taking into account all before mentioned results and restrictions, we can formulate the integer-to-integer IIR lifting steps [14] tat uses sixth order FIR with $\tilde{N}=6$,

$N = 6$, $B_p = 20, B_u = 8$, $C_p = 6, C_u = 6$ and special form of second order IIR filters with $a_1 = 0$ as following.

Analysis stage:

- prediction:

$$d_i^{(k)} = d_i^{(k-1)} + \left\lfloor \frac{A_2^p d_{i-2}^{(k)} - 108\left(s_{i-1}^{(k-1)} + s_{i+1}^{(k-1)}\right) + 23.3\left(s_{i-3}^{(k-1)} + s_{i+3}^{(k-1)}\right) - 3.3\left(s_{i-5}^{(k-1)} + s_{i+5}^{(k-1)}\right) + \dots}{256} \right\rfloor \quad (17)$$

- update:

$$s_i^{(k)} = s_i^{(k-1)} + \left\lfloor \frac{A_2^u s_{i-2}^{(k)}}{256} + \frac{56\left(d_{i-1}^{(k)} + d_{i+1}^{(k)}\right) + 9.3\left(d_{i-3}^{(k)} + d_{i+3}^{(k)}\right) - 1.3\left(d_{i-5}^{(k)} + d_{i+5}^{(k)}\right) - \dots}{256 - A_2^u} \right\rfloor \quad (18)$$

In formulas (17),(18), $\lfloor \cdot \rfloor$ denotes the operation of rounding to the nearest lower integer value.

A_2^p, A_2^u are adjusted in such a manner that the filters (17), (18) match to the spectral properties of the image data to minimize the well known first order entropy of the wavelet coefficients [14]

$$\min_{\vec{A}^p, \vec{A}^u} \left\{ H(\vec{d}) = -\sum_i p_i \log(p_i) \right\}, \quad (19)$$

where p_i denotes the probability of the different values of wavelet coefficients $\vec{d}$. More specifically, this criterion must be modified to take into account the method of wavelet coefficient codification. Usually, to code the wavelet coefficients, some zero-free method, such as EZW, SPIHT etc. is used. It means that the total quantity of information defined by the first order entropy is reduced by $p_0 \log(p_0)$, and it makes DWT-based coders to be an efficient data compression tool. Thus, A_2^p, A_2^u must be adjusted to achieve the minimum of the modified entropy at each k-th level of DWT decomposition

$$\min_{A_2^{p(k)}, A_2^{u(k)}} \left\{ \widetilde{H}(d^{(k)}) = -\sum_{i>1} p_i \log(p_i) \right\}; \quad (20)$$

Thus, one can obtain the optimal solution finding

$$\min_{\left\{A_2^{p(k)}\right\}\left\{A_2^{u(k)}\right\}} \left\{ \sum_{k=1}^{K-1} \widetilde{H}(\vec{d}^{(k)}) + H(\vec{d}^{(K)}) + H(\vec{s}) \right\}; \quad (21)$$

where K denotes the number of DWT decompositions, $H(\vec{d}^{(K)})$ is an entropy of wavelet coefficients at the highest level of decomposition, and $H(\vec{s})$ is an entropy of resulted approximations. Unfortunately, it is difficult to obtain an analytical solution for the optimal $\left\{A_2^{p(k)}\right\}, \left\{A_2^{u(k)}\right\}$ and one can find these values by simulations.

4 Experimental Results

The described in the previous section algorithm were tested on a set of 512x512 standard images "Lena", "Baboon", "Barbara", "Boats", "Goldhill", "Peppers", "Bridge" shown in Fig. 1 (these images are available, for example, at http://sipi.usc.edu/database/).

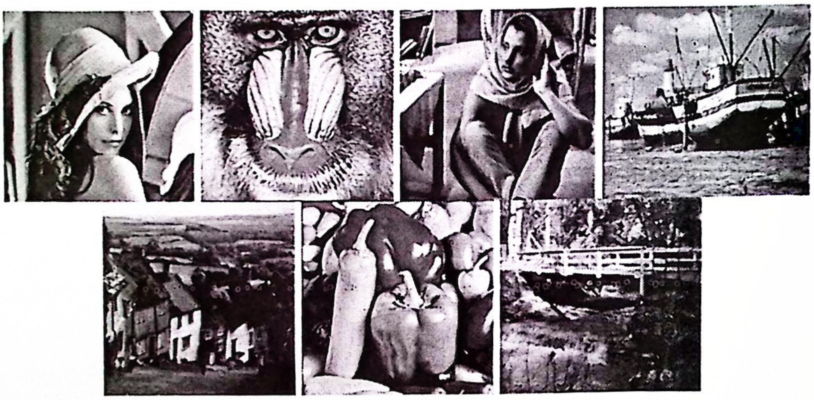

Fig. 1. Set of standard test images: "Lena", "Baboon", "Barbara", "Boats", "Goldhill", "Peppers", "Bridge"

Table presents the entropy values in bits per pixel (bpp) obtained for these images by applying standard lifting decomposition (1) – (3) and CDF(1,1) wavelet (Haar wavelet) with $\tilde{N}=1$, $N=1$, $B_p=0$, $B_u=0$, CDF(2,2) wavelet with $\tilde{N}=2$, $N=2$, $B_p=16$, $B_u=8$ (this wavelet is used by JPEG2000 for lossless image compression), and IIR lifting (17),(18) with optimal A_2^p, A_2^u values. In this paper, we assume that $\left\{A_2^{p(k)} = A_2^p, k=\overline{1,K}\right\}$, $\left\{A_2^{u(k)} = A_2^u, k=\overline{1,K}\right\}$. In al cases, the number of decomposition level was $K=6$.

Analyzing the simulation results presented in Table, one can conclude that the proposed IIR lifting transform performs better, providing lower entropy values for all test images in comparison to the FIR lifting. Varying IIR coefficients A_2^p, A_2^u, one can obtain higher data compression. In all cases, the IIR technique gives the best compression results.

Table. Entropy values in bpp for different lifting filters.

Technique	Image						
	Baboon	Lena	Barbara	Boat	Bridge	Peppers	Goldhill
CDF(1,1) lifting	5.896	3.803	4.566	4.044	4.232	4.237	4.427
CDF(2,2) Lifting	5.876	3.771	4.404	4.036	4.231	4.231	4.424
Proposed FIR only							
$A_2^p = 0, A_2^u = 0$	5.877	3.781	4.365	4.044	4.240	4.24	4.434,B_p=20 4.422,B_p=8
Proposed IIR Lifting	5.866	3.761	4.343	4.016	4.222	4.205	4.423,B_p=20 4.418,B_p=8
Optimal A_2^p	34	18	36	40	16	-26	17
Optimal A_2^u	11	12	12	2	11	17	-4

5 Conclusions

The algorithm of data-dependent DWT based on the generalized IIR lifting scheme was presented. The proposed algorithm requires only two additional integer sums and one floating point multiplication per pixel in comparison to the standard lifting decomposition. The presented results show that the derived algorithm provides lossless image compression and higher data compression rate comparing to the standard wavelet lifting technique. One can expect even better energy compaction, and, thus, higher compression rate with the presented algorithm using IIR filters of higher order and optimizing the IIR filter coefficients $\{A_2^{p(k)}\}$, $\{A_2^{u(k)}\}$ at each level of decomposition and FIR filter parameters $\tilde{N}^{(k)}$, $N^{(k)}$, $B_p^{(k)}, B_u^{(k)}$, $C_p^{(k)}, C_u^{(k)}$ for each k -th level as well. This aspect is a subject of future work.

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